MA719: Vector Space Methods in System Optimization – Optima and Equilibria

Instructor: Patrick L. Combettes, plc@math.ncsu.edu

Term: Fall 2025

Time: Tuesdays and Thursdays, from 15:00 to 16:15

Office hours: By appointment

Course objectives: The goal of modern optimization is the analysis and the construction of equilibria such as minimizers of functionals, saddle points, solutions to variational inequalities and monotone inclusions, solutions to minimax problems and games. This course provides an account of the essential tools of modern optimization in the setting of Banach and Hilbert spaces. The theoretical foundations will be developed from first principles and in tight interactions with application domains such as control theory, signal processing, game theory, partial differential equations, statistics, mechanics, inverse problems, machine learning, finance, and optimal transportation.

Prerequisite: Elementary knowledge of normed vector spaces.

Content: Theory and applications in the above areas will be intertwined. Main topics:

- Motivations
- Banach and Hilbert space
- Convex analysis
- Variational and minimization principles
- Regularization of minimization problems
- Notions of well-posedness
- Parametric duality in minimization problems
- Fixed point principles (Schauder, Brower, Kakutani-Fan-Glicksberg, Browder-Gohde-Kirk)
- Ky Fan inequality, Minimax theory
- Monotone operator theory
- Kuhn-Tucker theory for monotone inclusions
- Variational inequalities
- Nash equilibria and games
- Equilibrium finding dynamics and algorithms

Grading: HW 50%, project 50%.

Reference material (no purchase necessary):

- J.-P. Aubin, *Optima and Equilibria – An Introduction to Nonlinear Analysis,* second edition. Springer-Verlag, New York, 1998.
- H. H. Bauschke and P. L. Combettes, *Convex Analysis and Monotone Operator Theory in Hilbert Spaces,* 2nd ed. corrected printing. Springer, New York, 2019.
- R. Laraki, J. Renault, and S. Sorin, *Mathematical Foundations of Game Theory.* Springer, New York, 2019.
- R. Lucchetti, *Convexity and Well-Posed Problems.* Springer, New York, 2006.
- T. Rockafellar, *Conjugate Duality and Optimization.* SIAM, Philadelphia, PA, 1974.
- R. E. Showalter, *Monotone Operators in Banach Space and Nonlinear Partial Differential Equations*. Amer. Math. Soc., Providence, RI, 1997.
- C. Zălinescu, *Convex Analysis in General Vector Spaces.* World Scientific Publishing, River Edge, NJ, 2002.