Course title: *MA716 – Advanced Functional Analysis*

Instructor: Patrick L. Combettes, plc@math.ncsu.edu

Term: Fall 2026.

Time: Tuesdays and Thursdays, 16:30–17:45, SAS1218

Office : SAS 3276.

Office hours: By appointment.

Course objectives: This course provides an account of essential tools in modern functional analysis in view of its applications to approximation theory, optimization, the calculus of variations, partial differential equations, game theory, inverse problems, machine learning, statistics, and control. Duality and nonlinear analysis techniques are emphasized.

Prerequisite: MA426 or equivalent.

Content:

- Introduction: Notation, set theory, extended real line, nets, functions.
- Vector space: algebraic interior, absorption and balancedness, separation and duality, sets associated with functions and functions associated with sets, sublinearity, convexity, seminorms, Minkowski gauges.
- Topology: topological spaces, metrizability, sets and functions in topological spaces, semicontinuity, sequential spaces.
- Topological vector spaces: construction, duality, locally convex spaces, topology induced by a family of seminorns, separation, weak and weak$*$ topologies, inductive limits, sets and functions in topological vector spaces.
- Convex analysis in locally convex spaces, conjugate duality, perturbation duality, variational analysis.
- Theory of distributions: construction and basic operations, topological considerations, convolution, differentiation, Fourier transform.
- Fréchet spaces: construction, properties, examples.
- Reflexive Banach spaces: characterizations, examples, variational principles.

Grading: Homework 50%, final project 50%.

Reference material (no purchase necessary):

- H. H. Bauschke and P. L. Combettes, *Convex Analysis and Monotone Operator Theory in Hilbert Spaces,* 2nd ed. Springer, New York, 2017.
- T. Rockafellar, *Conjugate Duality and Optimization.* SIAM, Philadelphia, PA, 1974.
- W. Rudin, *Functional Analysis*, 2nd. ed. McGraw-Hill, New York, 1991.
- H. H. Schaefer, *Topological Vector Spaces*, 2nd. ed. Springer, New York, 1999.
- L. Schwartz, *Théorie des Distributions,* 2nd ed. Hermann, Paris, 1966.
- F. Trèves, *Topological Vector Spaces, Distributions, and Kernels.* Academic, New York, 1967.
- C. Zălinescu, *Convex Analysis in General Vector Spaces.* World Scientific Publishing, River Edge, NJ, 2002.