Course title: *MA493/MA591 – Introduction to Hilbert Spaces*

Instructor: Patrick L. Combettes, plc@math.ncsu.edu

Term: Fall 2024

Time: Tuesdays and Thursdays, from 15:00 to 16:15

Office: SAS 3276

Office hours: Tuesdays and Thursdays, from 16:30 to 17:30 (or anytime by appointment)

Prerequisite: MA242, MA305 or equivalent

Course objectives: This course provides an introduction to the basic theory of Hilbert spaces and to some of its current applications.

Course description: Hilbert space theory constitutes the core around which most of modern analysis has grown, with a wealth of applications in areas such as variational analysis, differential equations, signal processing, machine learning, harmonic analysis, control, inverse problems, and statistics. A Hilbert space is a powerful extension of the basic notion of a Euclidean plane, from which it retains many intuitive concepts such as orthogonality, parallelogram and polarization identities, orthogonal coordinate decompositions, projections and best approximation concepts, as well as linear vector transformations.

Content: The Euclidean plane, vector spaces, scalar products, pre-Hilbert spaces, projection theorem, orthonormal systems, Gram-Schmidt orthonormalization procedure, Hilbert spaces, orthonormal bases, Fourier series, linear operators on Hilbert spaces, linear equations in Hilbert spaces. Applications to differential equations, signal processing, optimization, physics, machine learning, inverse problems, and statistics.

Grading: Homework 30%, 2 in-class exams 30%, take-home final exam 40%.

Reference material (no purchase necessary, these are just general references):

- L. Debnath and P. Mikusinski, *Introduction to Hilbert Spaces with Applications*, 3rd. ed. Elsevier, 2000.
- N. Young, *An Introduction to Hilbert Space*. Springer, 1988.